FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2024
FULL TEST – III
PAPER –2
TEST DATE: 08-02-2024

ANSWERS, HINTS & SOLUTIONS

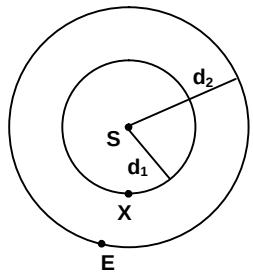
Physics

PART – I

SECTION – A

1. C

Sol. d_1 – distance between sun and cloud
d_2 – distance between sun and earth



P_S = Power of sun = $1.26 \times 10^3 \times 4\pi \times d_2^2$

Let the radius of particle is r so in equilibrium

$P_{emission} = P_{absorption}$

$$4\pi r^2 \sigma T^4 = \pi r^2 \times \frac{P_S}{4\pi d_1^2}$$

$$4\sigma T^4 = \frac{1}{4\pi d_1^2} \times 1.26 \times 10^3 \times 4\pi d_2^2$$

$$d_1 = \frac{d_2}{2T^2}\sqrt{\frac{1.26\times10^3}{5.6\times10^{-8}}} = \frac{1.5\times10^{11}}{2\times4\times10^6}\times15\times10^4 \simeq 2.81\times10^9\ \text{m}$$

2. C

Sol. Let at any instant separation between plates of capacitor *B* is *x*, then

$$C_{eq} = \frac{\frac{a\varepsilon_0}{d}\cdot\frac{a\varepsilon_0}{x}}{\frac{a\varepsilon_0}{d}+\frac{a\varepsilon_0}{x}} = \frac{a\varepsilon_0\ (dx)}{dx\ (d+x)} = \frac{a\varepsilon_0}{d+x}$$

$$Q = C_{eq}V = \frac{a\varepsilon_0}{d+x}V$$

$$\frac{dQ}{dt} = -\frac{a\varepsilon_0}{(d+x)^2}V\frac{dx}{dt} = -\frac{a\varepsilon_0}{(d+x)^2}Vv$$

Rate of work done on the battery $= -\left(\frac{dQ}{dt}\right)V = \frac{a\varepsilon_0 vV^2}{9d^2}$ (when $x = 2d$)

3. B

Sol. Let y be the height of liquid at some instant, then $-\frac{dy}{dt}$ = constant (given)

From equation of continuity $(\pi x^2)\left(-\frac{dy}{dt}\right) = a\sqrt{2gy}$ (a → area of hole)

Squaring the equation $y = kx^4$

4. A

Sol. $q[v_x\hat{i} + v_y\hat{j}] \times B_0\hat{k} - \alpha[v_x\hat{i} + v_y\hat{j}] = m\frac{d\vec{v}}{dt}$ ... (i)

$\therefore \quad a_y = -\frac{1}{m}\left[qv_xB_0 + \alpha v_y\right]$

and $a_x = \frac{1}{m}[qB_0v_y - \alpha v_x]$ ... (ii)

$\therefore$ At $t = 0$, $v_y = v_0$ and $v_x = 0$ and $t = t$, $v_x = 0$, $v_y = 0$

$\therefore \quad v_0 = \frac{1}{m}\left[qB_0x + \frac{\alpha^2 x}{qB_0}\right] \Rightarrow x = \frac{qB_0mv_0}{\alpha^2 + q^2B_0^2} = 10\text{m}$

5. A, B, C, D

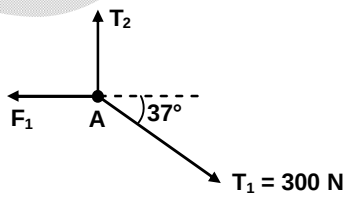
Sol. $T_2 = T_1\sin 37°$ ...(i)

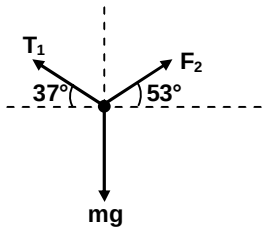
$F_1 = T_1\cos 37°$ ...(ii)

$F_2 = T_1\cos 37°/\cos 53°$ ...(iii)

$Mg = T_1\sin 37° + F_2\sin 53°$ ...(iv)

From the above equation find F_1, F_2, T_2 and mass of the block





6. B, C

Sol. $v^2 = v_0^2 + 2\left(\frac{qB_0}{m}\right)x_0$

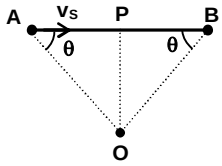
$v = \sqrt{2}v_0$

$a_n = \frac{qE_0}{m}\frac{v_0}{\sqrt{v_x^2 + v_0^2}}$

$R = \frac{v^2}{a_n} = \frac{[m^2v_0^2 + 2qE_0mx_0]^{\frac{3}{2}}}{qE_0v_0m^2} = 4\sqrt{2}\,x_0$

7. A, C

Sol. $f = f_0\left(\frac{v}{v + v_s\cos\theta}\right)$



$f_{min} = f_0\left(\frac{v}{v + v_s}\right)$

$1800 = 2000\left(\frac{300}{300 + v_s}\right)$

v_S = 33.33 m/sec

$f = f_0\left(\frac{v}{v - v_s\cos\theta}\right) \Rightarrow f_{max} = f_0\left(\frac{v}{v + v_S}\right)$ = 2250 Hz

SECTION – B

8. 27

Sol. $P_1 = \frac{4T}{r_1}$, $P_2 = \frac{4T}{r_2}$.

Thus $\frac{P_1}{P_2} = 3 = \frac{r_2}{r_1}$ or $r_2 = 3r_1$

Therefore, ratio of the volumes is

$$\frac{V_1}{V_2} = \frac{(4\pi/3)r_1^3}{(4\pi/3)r_2^3} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{1}{3}\right)^3 = \frac{1}{27}$$

9. 500

Sol. Intensity of the source at the cross-section *A*

$$\Rightarrow \qquad I = \frac{P}{4\pi(50r)^2} = \frac{1.25\times10^3}{4\pi(50r)^2}$$

Power absorbed by the end A

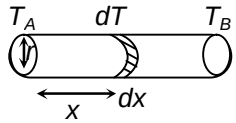
= 80% of $\left\{I \times \pi r^2\right\}$

$$= \frac{80}{100}\times\left(\frac{1.25\times10^3}{\pi\times10^4 r^2}\right)\left(\pi r^2\right) = 0.1\,\text{J/s}$$

Using Wien's displacement law, to determine the temperature of the end *B*

$\lambda_m T_B = 0.003$

$$T_B = \frac{0.003}{100000\times10^{-10}} = 300\,\text{K}$$



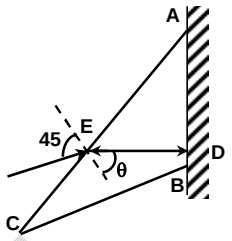
$$\left(\frac{dQ}{dt}\right) = -\left(\frac{T}{T_A}\right)\left(\pi r^2\right)\frac{dT}{dx}$$

$$\int_0^l \frac{0.1\,T_A.dx}{4\times10^{-4}} = -\int_{T_A}^{T_B} T dT$$

$\Rightarrow \quad T_A = 500$ K

10. 3

Sol. As incident ray retraces its path, the ray is incident normally on the silvered face.



From Snell's law at surface AC

$$\frac{\sin 45}{\sin r} = \sqrt{2}$$

$$\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = \sin r$$

r = 30°

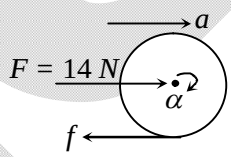
in ΔAED

$\angle A + \angle(90-30) = 90°$

$\angle A = 30°$

11. 4

Sol. $f_e = \mu mg = 100N$



$$F - f = ma$$

$$fR = I\alpha \Rightarrow f = \frac{2}{5}ma$$

$$\therefore \quad F = \frac{7}{5}ma \Rightarrow \frac{7}{5} \times 20 \times a = 14 \Rightarrow a = 0.5 \ m/s^2$$

$$\therefore \quad f = \frac{2}{5} \times 20 \times 0.5 = 4\ N$$

Since no external force acts on system, velocity of centre of mass remains constant.

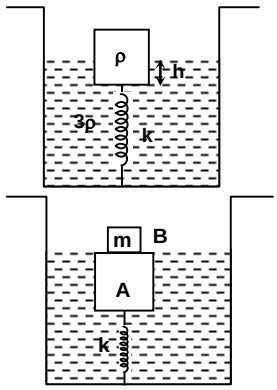
12. 8

Sol. In equilibrium

$(3\rho hg)A = \rho(Aa)g$

h = a/3

when block 'B' put on block 'A'



$$mg + (a^3\rho)g = k\left(\frac{2a}{3}\right) + (a^3 3\rho)g$$

$$m = 2a^3\rho + \frac{2kq}{3g} = 2a^3\rho + \frac{2}{3}a^3\rho$$

$$m = \frac{8}{3}a^3\rho = \frac{8}{3}m_0$$

13. 9

Sol. COLM

$$2mv_0 - mv_0 = 2mv$$

$$v = \frac{v_0}{2}$$

COME

$$-\frac{Gm^2}{a} + \frac{mv_0^2}{2} + \frac{m(2v_0)^2}{2} = -\frac{Gm^2}{r} + \frac{2m}{2}\left(\frac{v_0}{2}\right)^2$$

$$-\frac{Gm^2}{a} + \frac{mv_0^2}{2}\left[1 + 4 - \frac{1}{2}\right] = -\frac{Gm^2}{r}$$

$$-\frac{Gm^2}{a}+\frac{mv_0^2}{2}\left[\frac{9}{2}\right]=-\frac{Gm^2}{r}$$

$$\frac{Gm^2}{a}-\frac{9}{4}mv_0^2=\frac{Gm^2}{r}$$

$$r=\frac{4Gma}{(4Gm-9v_0^2a)}$$

k = 9

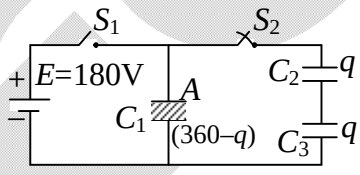
SECTION – C

14. 360.00

Sol. When S_1 is closed and S_2 is open,
Charge on $A = C_1E = 2 \times 180 = 360\ \mu C$

15. 144.00

Sol. Now S_1 opened and S_2 closed



$$\frac{360-q}{C_1}=\frac{q}{C_2}+\frac{q}{C_3}$$

$$720-2q=3q$$

q = 144 μC

16. 0.50

Sol. The emission of photoelectron will stop when $\frac{hc}{\lambda_{violet}}=\phi+eV$, where V is the potential of sphere.

$$\frac{12408}{4136}=2.5+eV$$

$$V=0.5\,V$$

17. 1.39

Sol. Number of electrons emitted to raise the potential of sphere to 0.5V is

$$n=\frac{Q}{e}=\frac{4\pi\varepsilon_0 rV}{e}$$ = 3.47 × 10^6 electrons

Time taken = $\frac{3.44\times10^{-6}}{2.5\times10^{9}}=1.39\times10^{-3}$ s

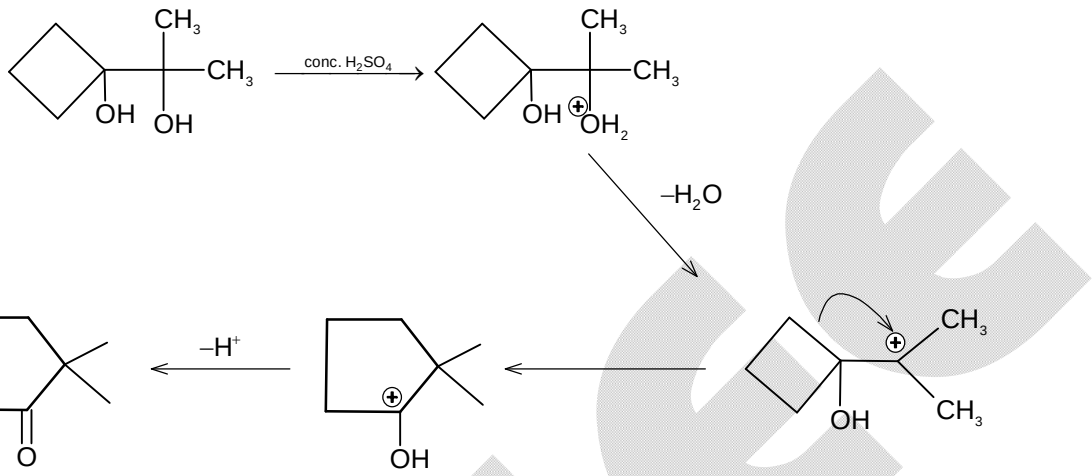
Chemistry

PART – II

SECTION – A

18. D

Sol.



19. D

Sol. Factual

20. B

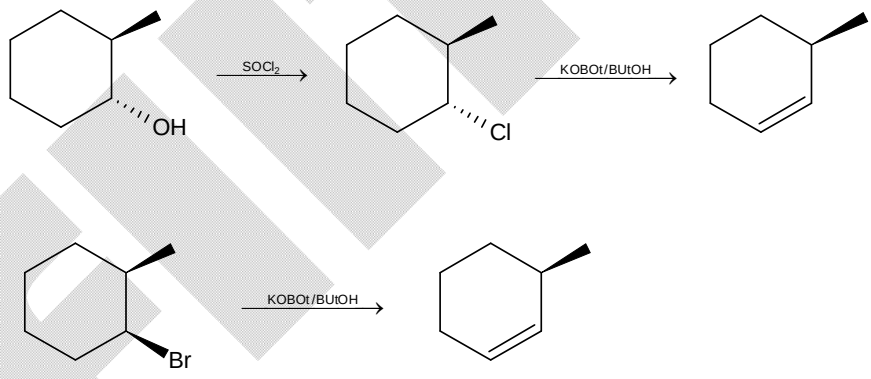
Sol. Factual

21. A

Sol. Factual

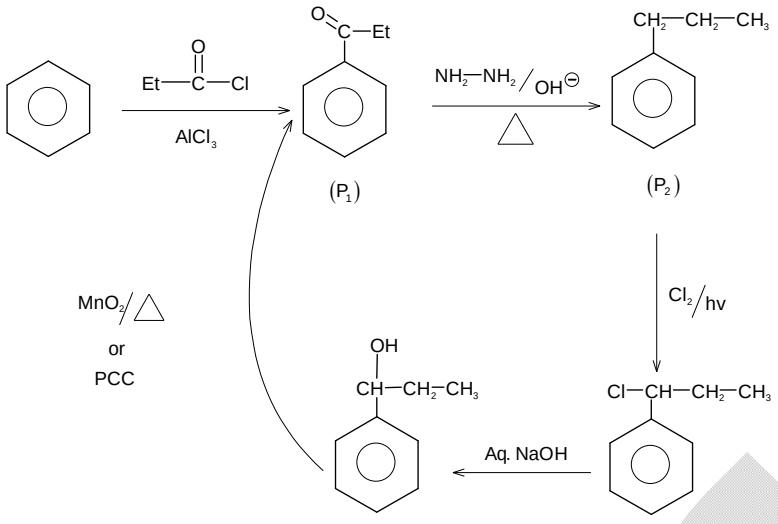
22. A, B

Sol.



23. A, B

Sol.



24. A, B, C

Sol. Factual

SECTION – B

25. 12

Sol. $$2.32 = \frac{4 \times M}{6.022 \times 10^{23} \left(1.221 \times 10^{-7}\right)^3}$$

$$M = \frac{2.32 \times 6.022 \times 10^{23} \times (1.221)^3 \times 10^{-21}}{4}$$

M = 635. 8 g/mol

635.8 = 204 + 27 + (96 × 2) + 18 x

$x \approx 12$

26. 2

Sol. $$\frac{2.6}{52} \times n = \frac{0.56}{22.4} \times 4$$

n = 2

27. 6

Sol. $$0.5 = 1.86 \times \frac{x \times 1000}{60 \times 372}$$

x = 6g

28. 8

Sol. $2^3 = 8$

29. 4

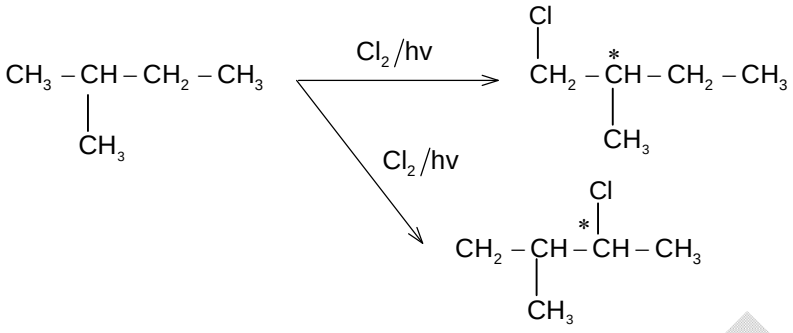
Sol. Mole of AgI = $\frac{11.62 \times 10^{-3}}{235} = 5 \times 10^{-5}$

Mole of A = $\frac{4.24 \times 10^{-3}}{329} = 1.28 \times 10^{-5}$

Number of methoxy groups present in A = $\frac{5\times10^{-5}}{1.28\times10^{-5}} = 4$

30. 2

Sol.



SECTION – C

31. 5.45

Sol. $pI = \frac{1.8+9.1}{2} = 5.45$

32. 9.75

Sol. $pI = \frac{9+10.5}{2} = 9.75$

33. 1200.00

Sol. $V_1 = 320\,mL$

$V_2 = 10\,mL$

$P_1 = 1\,atm$

$T_1 = 300K$

$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_2 = 300\times\left(\frac{320}{10}\right)^{7/5-1} = 1200k$

34. 243.00

Sol. $P_2 = P_1\left(\frac{320}{10}\right)^{7/5}$

$P_2 = 128\ atm$

$|W| = \frac{P_2V_2 - P_1V_1}{\gamma-1}$

$|W| = \left(\frac{128\times10\times10^{-3} - 1\times320\times10^{-3}}{7/5-1}\right)\times\frac{8.314}{0.0821} = 243J$

Mathematics

PART – III

SECTION – A

35. B

Sol. Total cases = $2 \times 3 \times 3 \times 2 = 36$

36. C

Sol. Consider the inequality $\sum x_i^2 \sum y_i^2 \geq \left(\sum x_i y_i\right)^2$

i.e. $\left[(2\sqrt{2})(\sqrt{2}a) + (3)(b) + (c)^2\right]^2 \leq (8+9+1)(2a^2+b^2+c^2)$

$\Rightarrow 2a^2+b^2+c^2 \geq \frac{49}{18}$ and least value is achieved when $\frac{2\sqrt{2}}{\sqrt{2}a} = \frac{3}{b} = \frac{1}{c} = \frac{1}{k}$ i.e.

$\Rightarrow a = 2k, b = 3k, c = k \Rightarrow k = \frac{7}{18}$

37. D

Sol. Let E be mid-point of AB

$3x^2 - 8x - 16 = 0$

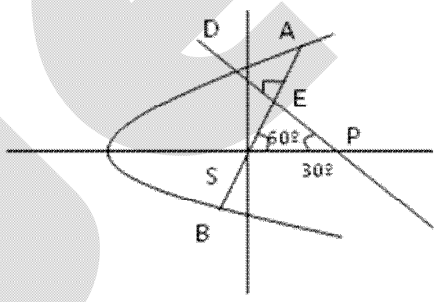
$x_1 + x_2 = \frac{8}{3}$

$\frac{x_1 + x_2}{2} = \frac{4}{3}$

$\therefore$ x-coordinate of E is 4 / 3

So SP = 2SE

$\therefore$ SE = 8/3 and SP = 16 / 3



38. C

Sol. Plane containing the line will be given as $(4x - 2y - 10) + \lambda (5y - 4y + 3) = 0$

$\Rightarrow 4x + (5\gamma - 2)y - 4\lambda z + 3\lambda - 10 = 0$ the plane passing through (4, 3, 7) will give λ

$\Rightarrow 16x + (5\gamma - 2)y - 4\lambda z + 3\lambda - 10 = 0$

$\Rightarrow -10\lambda = 0 \Rightarrow \lambda = 0$

i.e., $4x - 2y - 10 = 0$

If this rotated an angle $\frac{\pi}{2}$ the plane will be $4x + (5\lambda' - 2)y - 4\lambda' z + (2\lambda' - 10) = 0$

Also $16 + (5\lambda' - 2)(-2) = 0$

$\Rightarrow \lambda' = 2$; i.e. $4x + 8y - 8z = 4$

$\frac{x}{l} = \frac{y}{m} = \frac{z}{n} = \lambda \Rightarrow \overrightarrow{OA} + y\overrightarrow{OB} + z\overrightarrow{OC} = \vec{0}$

39. A, C

Sol. Clearly $a^2 + b^2 = k^2$

Again $\left(a + \frac{1}{b}\right)^2 + \left(b + \frac{1}{a}\right)^2 = a^2 + b^2 + \frac{1}{a^2} + \frac{1}{b^2} + 2\left(\frac{a}{b} + \frac{b}{a}\right)$

$AM \geq HM \Rightarrow \frac{\frac{1}{a^2} + \frac{1}{b^2}}{2} \geq \frac{2}{a^2 + b^2} \Rightarrow \frac{1}{a^2} + \frac{1}{b^2} \geq \frac{4}{k^2}$

$$\Rightarrow\left(a+\frac{1}{b}\right)^2+\left(b+\frac{1}{a}\right)^2\geq k^2+\frac{4}{k^2}+4$$

40. A, D

Sol. $r_i = i + 1, \forall\ i = 1, 2,....99$

41. A, B

Sol. $\frac{\sqrt{5}-1}{4}\times\frac{1}{\sin x}+\frac{\sqrt{10+2\sqrt{5}}}{4}\times\frac{1}{\cos x}=2$

$\frac{(\sqrt{5}-1)}{4}\cos x+\frac{\sqrt{10+2\sqrt{5}}}{4}\times\sin x=2\sin x.\cos x$

$\sin\left[x+\frac{\pi}{10}\right]=\sin 2x$

$x+\frac{\pi}{10}=2x,\ x=\frac{\pi}{10}$

or $\sin\left(x+\frac{\pi}{10}\right)=\sin(\pi-2x)$

$x+\frac{\pi}{10}=\pi-2x$

$3x=\pi-\frac{\pi}{10}=\frac{9\pi}{10},\ x=\frac{3\pi}{10}.$

SECTION – B

42. 3

Sol. Using LMVT, $\frac{\tan^{-1}\beta-\tan^{-1}\alpha}{\beta-\alpha}=\frac{1}{1+c^2}$

Where $0<\alpha<c\ \beta<\sqrt{3}$

So, $\frac{1}{4}<\frac{1}{1+c^2}<1$

So, $1<\frac{\beta-\alpha}{\tan^{-1}\beta-\tan^{-1}\alpha}<4$

So the answer is 3

43. 5

Sol. Let $x_i = \sin\theta_i$ [I = 1, 2, ……,10]

$\sum_{i=1}^{10}(\sin\theta_i)^3=0\Rightarrow 4\sum_{i=1}^{10}(\sin\theta_i)^3=0$

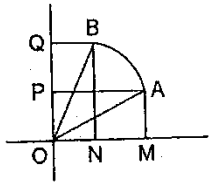
$\Rightarrow 3\sum_{i=1}^{10}\sin\theta_i-\sum_{i=1}^{10}\sin 3\theta_i=0$

$\sum_{i=1}^{10}\sin\theta_i=\frac{1}{3}\sum_{i=1}^{10}\sin 3\theta_i\leq\frac{10}{3}$

44. 3

Sol. Area of sector is $\frac{\pi}{8}$

Area (ABMN) = Area of sector + $\Delta_{OAM} - \Delta_{OBN}$



Area (ABPQ) = Area of sector + $\Delta_{OBQ} - \Delta_{OAP}$
Add for answer

45. 0

Sol. PQ and AB intersect inside the circle,
Let PR = a, RS = b, QS = c
$\therefore \quad a(b+c) = 3 \times 7$
Similarly $(a+b)c = 3 \times 7$
$\Rightarrow a = c$
$\therefore |PR - QS| = 0$

46. 0

Sol. $S_n = \sum_{k=0}^{n} (-4)^k \ {}^{n+k}C_{2k}$

$S_{n+1} = \sum_{k=0}^{n+1} (-4)^k \ {}^{n+k+1}C_{2k}$

$S_{n-1} = \sum_{k=0}^{n-1} (-4)^k \ {}^{n-k+1}C_{2k}$

Consider ${}^{n+1}C_r + {}^{n-1}C_r - 2^n C_r =$ coefficient of x^r in $\{(1+x)^{n+1} + (1+x)^{n-1} - 2(1+x)^n\}$
= coefficient of x^r in $(1+x)^{n-1}\{(1+x)^2 + 1 - 2(1+x)\}$
= coefficient of x^r in $(1+x)^{n-1}\{x + 2x + 2 - 2x - 2\}$
= coefficient of x^{r-2} in $(1+x)^{n-1} = {}^{n-1}C_{r-2}$
Put $n \to n+k$ and $r \to 2k$
$\Rightarrow {}^{n+k+1}C_{2k} + {}^{n+k-1}C_{2k} - 2\,{}^{n+k}C_{2k} = {}^{n+k-1}C_{2(k-1)}$.
Multiplying by $(-4)^k$ and applying summation on the both sides

$$\sum_{k=0}^{n+1} (-4)^k \ {}^{n+k+1}C_{2k} + \sum_{k=0}^{n-1} (-4)^k \ {}^{n+k-1}C_{2k} - 2\sum_{k=0}^{n} (-4)^k \ {}^{n+k}C_{2k} = \sum_{k=1}^{n+1} (-4)^k \ {}^{n+k-1}C_{2(k-1)}$$

(Note that the limits of summation are different for each term)

$$\Rightarrow S_{n+1} + S_{n-1} - 2S_n = \sum_{k=1}^{n+1} (-4)^k \ {}^{n+k-1}C_{2(k-1)}$$

Let $k - 1 = t$

$$\sum_{t=0}^{n} (-4)^t (-4)^1 \ {}^{n+t}C_{2t} = -4S_n$$

$S_{n+1} + 2S_2 + S_{n-1} = 0$
Put n = 2010.

47. 8

Sol. Replacing x by $\frac{2}{x}$ we get,

$$\left(\frac{8}{x^2} + \frac{8}{x} + 4\right)^{10} = \sum_{r=0}^{20} a_r \left(\frac{2}{x}\right)^r \Rightarrow 2^{10}(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r 2^r x^{20-r} \Rightarrow \sum_{r=0}^{20} a_r x^r = \sum_{r=0}^{20} a_r 2^{r-10} x^{20-r}$$

Comparing coefficient x^7 both sides, we get $a_7 = a_{13} \times 2^3$.

SECTION – C

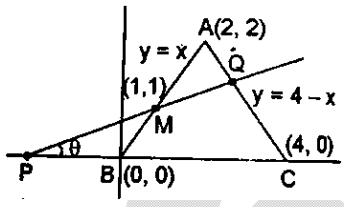
48. 1.00

49. 0.00

Sol. (Q.48 – 49):

Equation of line PM is $y - 1 = \tan\theta\,(x - 1)$

Coordinates of Q = $\left(\frac{3+\tan\theta}{1+\tan\theta}, \frac{1+3\tan\theta}{1+\tan\theta}\right)$



Area of $\Delta AMQ = \frac{1-\tan\theta}{1+\tan\theta}$

Area of quadrilateral BMQC

$$A = \frac{1}{2}\times 4\times 2 - \frac{1-\tan\theta}{1+\tan\theta} = \frac{3+5\tan\theta}{1+\tan\theta}$$

$$\tan\theta = \lim_{x\to\frac{\pi}{3}} \frac{\sin\left(\frac{\pi}{3}-x\right)}{2\cos x - 1} = \frac{1}{\sqrt{3}}$$

50. 18.00

51. 3.00

Sol. (Q.50 – 51):

$\vec{r} = x\hat{i} + y\hat{j}$

$x^2 + y^2 + 8x - 10y + 40 = 0$

$C = (-4, 5);\ r = 1$

$P_1 = \max\left\{(x+2)^2 + (y-3)^2\right\}$

$P_2 = \min\left\{(x+2)^2 + (y-3)^2\right\}$

$CP = 2\sqrt{2},$

Then $P_2 = \left(2\sqrt{2}-1\right)^2, P_1 = \left(2\sqrt{2}+1\right)^2$

$\overline{OA} = 2\hat{i} + 2\hat{j}$

$\overline{OB} = 3\hat{i}$

$\therefore \overline{AB}.\overline{OB} = 3.$